Exciting-Force Operators for Ship Propellers

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The unsteady transfer functions or vibratory-exciting force and moment operators for any ship propeller are found by a procedure which employs an existing computer program. These functions can then be applied to any given radial variation of a spatially varying hull wake or inflow to yield three forces, three moments, and the blade bending moments. With these operators (which can be found once and for all for a systematic series of propellers), the task of predicting the vibratory-exciting forces and moments acting on the bearings of a particular ship is reduced to a simple calculation that can be made with a desk calculator in a matter of minutes. Thus, once a complete set of operators are obtained from the elaborate computer program, subsequent applications to any specific wakes does not require the computer program. Application of the method to a 5-bladed propeller is given in a worked example using simple formulas and tabulated values of the operators.

Introduction

IT is a well-known feature of those mechanical systems which are amenable to linear analysis, that they may be viewed as "transformers" of an input into an output and that the output is a linear function of the input. Such systems can be completely specified by their normalized transfer functions or response operators. In the case of the motions of ships, the heave and pitch operators are the responses per unit wave height and per unit wave slope, respectively, as obtained from measurements or calculations for a selected set of wave encounter frequencies. The advantage of these normalized operators is that they can then be applied at any time to any specific set of wave conditions (for which linearity holds) without the further application of model tests or computers. The development of analogous operators for the quick calculation of the mean and vibratory forces generated by propellers operating in spatially variable inflows (steady wakes) is urgently needed. It is the purpose of this short paper to demonstrate how exciting-force operators for ship propellers can be found from an existing computer program which heretofore has only been applied to find answers for specific wakes.

Analysis

The resolution of all velocity components normal to any section of a propeller blade at radius r where the local blade pitch angle is ϕ_p leads to the equation:

$$u_q \cos\lambda + v_q \sin\lambda + \epsilon_q(\phi_p - \lambda)V + \iint \Delta p_q K dS = 0 \quad (1)$$

where λ is the hydrodynamic pitch angle defined by $\tan^{-1}(U/r\omega)$; ω is the angular velocity of the propeller; V is the resultant velocity $[U^2 + (r\omega)^2]^{1/2}$; U is the forward speed or freestream speed (unaffected by wake); u_q is the axial wake component of harmonic order q at the radius r; v_q is the tangential wake component of order q at radius r; Δp_q is the *unknown* pressure jump at any point on the blade area S; K is the influence function or unit pressure dipole which accounts for the flow induced by all other elements of the blade and all other blades; and $\epsilon_q = 1, q = 0$; $\epsilon = 0, q > 0$.

The last term in Eq. (1) represents the induced velocities developed by all of the unknown pressure jumps on the blade. The kernel function (or influence function) K is itself an integral over all the previous history of the fluid motion and thus embodies the unsteadiness of the motion induced by the blades.

Equation (1) has been reduced to a computer operation by Tsakonas, Jacobs et al.[1,2] which yields the pressure distribution Δp_q and, hence, the thrust, torque, transverse and vertical forces, as well as moments, about these axes for both the mean effects of the wake as well as the various harmonics. To date, some 30 cases have been treated. The results are particular to the propeller and specific wake. Here we wish to get at the propeller transfer functions themselves, which are characteristic of the propeller (and independent of the wake) as are the familiar thrust and torque coefficient curves as a function of J (advance ratio).

The equation is manipulated into a set of coupled linear algebraic equations which, when solved for the loading distribution (integral of pressure) over the chord at 8 radial positions along the radius, is

$$(l_q)_i = \sum_{j=1}^{8} (\alpha_q)_{ij} \{(u_q)_j \cos\lambda_j + (v_q)_j \sin\lambda_j + \epsilon_q[(\phi_p)_j - \lambda_j]V_j\} \quad (2)$$

where the α_{ij} are the elements of the inverse kernel matrix, i.e., $[K]^{-1}$ or influence functions. The first term in Eq. (12) is that proportional to the axial wake velocity of order q and the second is that proportional to the tangential components of the same orders.

These loadings, when multiplied by the appropriate factors to convert them to components of thrust, torque-producing forces, etc., may then be summed or integrated over the span of the blade to yield the total effect sought for the entire propeller.

If we perform this sum, then the result for the thrust coefficient arising from the qth spatial harmonic in the wake is expressed as a linear combination of the input velocities by

$$(K_x) = \sum_{j=1}^{8} (\gamma_q)_j \{(u_q)_j \cos\lambda_j + (v_q)_j \sin\lambda_j + \epsilon_q[(\phi_p)_j - \lambda_j]V_j\} \quad (3)$$

where γ_j is the sum of the $(\alpha_q)_{ij}$ as modified to give thrust. The coefficient $(\gamma_\nu)_j$ is the exciting force operator for thrust.

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Table 1 Principle characteristics of a 5-bladed ship propeller

Diameter	22.5 ft
Pitch-diameter ratio	1.088
Blade area ratio	0.564
Mean-width ratio	0.221
Design advance ratio J	0.993

To illustrate these ideas more explicitly, let us write Eq. (2) in a compact form by denoting w as the velocity normal to the j-blade sections as arises from the combination of axial and tangential wakes provided by the νth spatial harmonics of these components. Then Eq. (2) may be written out in matrix form to yield the local contributions to thrust at each radial blade location by first multiplying each *normal* loading $(l_\nu)_i$ by $\cos\lambda$ (thrust being the axial component of the normal loading). Then one has the array from subdivision of the blade span into 8 radial stations:

$$\begin{aligned} f_{x1} &= \alpha_{11}'w_1 + \alpha_{12}'w_2 + \alpha_{13}'w_3 + \ldots + \alpha_{18}'w_8 \\ {}_{x2} &= \alpha_{21}'w_1 + \alpha_{22}'w_2 + \alpha_{23}'w_3 + \ldots + \alpha_{28}'w_8 \\ &\;\vdots \\ f_{x8} &= \alpha_{81}'w_1 + \alpha_{82}'w_2 + \alpha_{83}'w_3 + \ldots + \alpha_{88}'w_8 \end{aligned} \tag{4}$$

(Here the primes indicate that the α_{ij} have been multiplied by $\cos\lambda$ at each station.)

Calculations made in the past have obtained the left sides by summing the products on the right side of Eq. (4) for specific wake values $w_1, w_2 \ldots w_8$. However, it is the sum of all the left-side thrust contributions (where multiplied by the common interval between the stations) which yields the total thrust at the particular harmonic of interest. Hence, the total thrust coefficient is obtained by summing the elements of the above array vertically to yield for the total axial force coefficient

$$K_x = \sum_{j=1}^{8} \alpha_{j1}'w_1 + \sum_{j=1}^{8} \alpha_{j2}'w_2 + \ldots \sum_{j=1}^{8} \alpha_{j8}'w_8 \tag{5}$$

The coefficients of the normal velocities as exhibited by the sums of the α' are the exciting force operators for thrust. They are functions only of blade geometry, pitch diameter ratio, blade area ratio and advance coefficient J. They are completely independent of the wake. They can be found from the existing computer program by successive insertions of 8 wakes, each time all but one of the wakes being zero and the wake at the remaining location being unity. The sums of the α' are thus simply partial derivatives of the total thrust with respect to the normal velocity at a particular element. (Or simply thrust per unit wake input at a radial station). Thus, for station 2 with $w_2 = 1.0$ and all other w's being zero,

$$\sum_{j=1}^{8} \alpha_{j2}' = \frac{(K_{F_x})_2}{(1.0)} = \frac{\partial K_{F_x}}{\partial w_2} = K_{x2}' \tag{6}$$

Hence, the total thrust is expressible by

$$K_x = \sum_{\nu=1}^{8} K_{x\nu}'w_\nu \tag{7}$$

where

$$K_{x\nu}' = \sum_{j=1}^{8} \alpha_{j\nu}' \tag{8}$$

The exciting-force operators or partial derivatives at each radial location are complex numbers composed of a real (or inphase) and an imaginary (out-of-phase)† component. The normal wake velocity is also complex, made up of a cosine (inphase) and a sine (out-of-phase) function of the qth multiple of the blade position angle. The real and imaginary parts of these products must be found. The amplitude is then the square root of the sum of the squares of these parts and the tangent of the phase angle is the ratio of the imaginary to the real part.

The same type of relations exist for each of the force and moments generated on a propeller in a wake.[3] The total ensemble of these operators, one set for each force and moment, constitute a complete force and moment characterization of the propeller. Once they are provided, the set of vibratory forces and moments and blade bending moments can be obtained by appropriate multiplication of the operators for each with a given wake. Thus, results for any specified wakes can be secured in a matter of minutes by the use of a desk calculator.

In the example provided in the next section, the application of a set of operators to a specific wake is illustrated and the basic formulas for the combination of operators and wakes are clearly depicted.

An Example of the Application of Exciting-Force Operators

The force and moment exciting-force operators for a 5-bladed propeller are listed in Tables 2 and 3. The principal characteristics of the propeller are given in Table 1.

The wake velocities in fraction of the forward speed for a particular single screw ship are listed in Table 4. The longitudinal wake only contains cosine terms, i.e., $a_n \cos n\theta$, θ being the blade angle. (This is due to the fact that the ship is symmetrical about the centerplane.) The tangential velocity is composed of $B_n \sin n\theta$ (since this must be an odd function of blade angle for a single screw hull).

The velocity normal to the blade section is, in general, given by

$$w_n(r) = [a_n(r)\cos\lambda(r) + A_n \sin\lambda(r)] - i[b_n(r)\cos\lambda(r) + B_n(r)\sin\lambda(r)] \tag{9}$$

which, for a single screw ship where $A_n = 0$, $b_n = 0$, reduces to

$$w_n(r) = a_n \cos\lambda - iB_n \sin\lambda \tag{10}$$

To obtain these values, Table 5 is constructed.

If we designate the real (R) or inphase and the imaginary (I) or out-of-phase operators by the notation $\tilde{K}[(\;)']_R$ and $\tilde{K}[(\;)']_I$, where the inner subscript designates the particular force or moment, then, for example, for vibratory axial force, it is necessary to sum the products

$$\tilde{K}_x(q) = \sum_{\nu=1}^{8} [(\tilde{K}_{x\nu}')_R + i(\tilde{K}_{x\nu}')_I][a_q(\nu)\cos\lambda_\nu - iB_q(\nu)\sin\lambda_\nu]\ddagger \tag{11}$$

or

$$\tilde{K}_x(q) = \sum_{\nu=1}^{8} [\underset{\text{T5-6}}{a_q(\nu)\cos\lambda_\nu}\underset{\text{T2-2}}{(\tilde{K}_{x\nu}')_R} + \underset{\text{T5-13}}{B_q(\nu)\sin\lambda_\nu}\underset{\text{T2-3}}{[(\tilde{K}_{x\nu}')_I]}] + i[\underset{\text{T5-6}}{a_q(\nu)\cos\lambda_\nu}\underset{\text{T2-3}}{(\tilde{K}_{x\nu})_I} - \underset{\text{T5-13}}{B_q(\nu)\sin\lambda_\nu}\underset{\text{T2-2}}{(\tilde{K}_{x\nu})_R}] \tag{12}$$

† By out-of-phase component is meant the quadrature component.

‡ Wake is for single screw ship. For general case, the wake input should read $(q_q \cos\lambda + A_q \sin\lambda) - i(b_q \cos\lambda + B_q \sin\lambda)$.

Table 2 Nondimensional exciting-force operators at 8 radial locations along blades of a 5-bladed propeller for relevant harmonic orders q[a,b]

Radial station ν	r/r_0	Longitudinal axis, Mean: $\bar{K}_x' \times 10^3$, $q = 0$	Longitudinal axis, Vibratory: $\tilde{K}_x' \times 10^3$, $q = 5$		Transverse axis, Mean: $\bar{K}_y' \times 10^3$, $q = 1$		Transverse axis, Vibratory: $\tilde{K}_y' \times 10^3$, $q = 4$		$q = 6$	
Column no. →		1 real	2 real	3 imag.	4 real	5 imag.	6 real	7 imag.	8 real	9 imag.
1	0.25	22.76	−2.952	8.737	8.254	3.105	0.414	5.052	−3.443	3.261
2	0.35	42.78	−4.851	18.82	13.96	5.106	1.518	8.612	−4.993	6.586
3	0.45	61.25	−4.106	29.97	17.95	6.178	3.355	11.29	−5.300	9.641
4	0.55	77.25	6.762	39.87	20.40	5.827	7.257	12.74	−3.130	12.19
5	0.65	90.10	25.75	43.77	21.27	4.597	11.60	12.09	1.185	12.50
6	0.75	102.0	53.88	39.09	21.29	2.791	15.75	9.252	7.100	10.04
7	0.85	155.3	133.9	56.63	30.47	1.662	30.16	9.641	22.86	15.33
8	0.95	89.42	87.89	−8.236	16.67	−0.508	17.84	−1.321	16.22	−0.727

[a] Vertical force operators are found from the relation $\tilde{K}_z' = i\tilde{K}_y'$.
[b] The value of $\bar{K}_x$ at station 5 is 0.02575, i.e., all values must be multiplied by 10^{-3}.

where the location of the required quantities are indicated, for example, T5-6 is Table 5, column 6. Upon summing over the station numbers $\nu = 1$–8, one obtains the real and imaginary parts of the total axial vibratory force at harmonic order q. This is then of the form

$$\tilde{K}_x(q) = [\tilde{K}_x(q)]_R + i[\tilde{K}_x(q)]_I \quad (13)$$

so that the amplitude and phase of this force are given by

$$|\tilde{K}_x(q)| = \{[\tilde{K}_x(q)]_R + [\tilde{K}_x(q)]_I\}^{1/2} \quad (14)$$

$$\phi = \tan^{-1}\{[\tilde{K}_x(q)]_I/[\tilde{K}_x(q)]_R\} \quad (15)$$

where

$$[\tilde{K}_x(q)]_R = \sum_{\nu=1}^{8} [a_q(\nu)\cos\lambda_\nu(\tilde{K}_{x\nu}')_R + B_q(\nu)\sin\lambda_\nu(\tilde{K}_{x\nu}')_I] \quad (16)$$

$$[\tilde{K}_x(q)]_I = \sum_{\nu=1}^{8} [a_q(\nu)\cos\lambda_\nu(\tilde{K}_{x\nu}')_I - B_q(\nu)\sin\lambda_\nu(\tilde{K}_{x\nu}')_R] \quad (17)$$

Detailed numerical evaluation of Eq. (17) and hence Eqs. (14) and (15) can be found in Table 6.

The vibratory torque or moment about the propeller axis is found in the same way using columns 2 and 3 of Table 3 in lieu of the same columns in Table 2.

The transverse and vertical forces require the sum of the products of the operators and wake harmonics at the order q-1 and those products for order $q + 1$. Thus, the expression for the transverse force is

$$\tilde{K}_y(q) = \sum_{\nu=1}^{8} \{\tilde{K}_{y\nu}'(q-1)w_{q-1}(\nu) + K_{y\nu}'(q+1)w_{q+1}(\nu)\} \quad (18)$$

where all the quantities are complex. For the average or mean lateral force $q = 0$, and Eq. (18) reduces to

$$\bar{K}_y(0) = \sum_{\nu=1}^{8} \bar{K}_{\nu y}'(1)w_1(\nu) \quad (19)$$

The real and imaginary parts of Eq. (18) are found to be

$$[\tilde{K}_y(q)]_R = \sum_{\nu=1}^{8} \{\underset{\text{T5-5}}{a_{q-1}}\cos\lambda\underset{\text{T2-6}}{[\tilde{K}_{y\nu}'(q-1)]_R} + \underset{\text{T5-12}}{B_{q-1}}\sin\lambda\underset{\text{T2-7}}{[\tilde{K}_{y\nu}'(q-1)]_I} + \underset{\text{T5-7}}{a_{q+1}}\cos\lambda\underset{\text{T2-8}}{[\tilde{K}_{y\nu}'(q+1)]_R} + \underset{\text{T5-14}}{B_{q+1}}\sin\lambda\underset{\text{T2-9}}{[\tilde{K}_{y\nu}'(q+1)]_I}\} \quad (20)$$

$$[\tilde{K}_y(q)]_I = \sum_{\nu=1}^{8} \{\underset{\text{T5-5}}{a_{q-1}}\cos\lambda\underset{\text{T2-7}}{[\tilde{K}_{y\nu}'(q-1)]_I} - \underset{\text{T5-12}}{B_{q-1}}\sin\lambda\underset{\text{T2-6}}{[\tilde{K}_{y\nu}'(q-1)]_R} + \underset{\text{T5-7}}{a_{q+1}}\cos\lambda\underset{\text{T2-9}}{[\tilde{K}_{y\nu}'(q+1)]_I} - \underset{\text{T5-14}}{B_{q+1}}\sin\lambda\underset{\text{T2-8}}{[\tilde{K}_{y\nu}'(q+1)]_R}\} \quad (21)$$

Table 3 Nondimensional exciting-moment operators at 8 radial locations along blades of a 5-bladed propeller for relevant harmonic orders q[a]

Radial station ν	r/r_0	Moment about long'l. axis, Mean: $\bar{K}_{Qx}' \times 10^3$, $q = 0$	Moment about long'l. axis, Vibratory: $\tilde{K}_{Qx}' \times 10^3$, $q = 5$		Moment about transverse axis, Mean: $\bar{K}_{Qy}' \times 10^3$, $q = 1$		Moment about transverse axis, Vibratory: $\tilde{K}_{Qy}' \times 10^3$, $q = 4$		$q = 6$	
Column no. →		1 real	2 real	3 imag.	4 real	5 imag.	6 real	7 imag.	8 real	9 imag.
1	0.25	−3.756	0.468	−1.430	1.752	0.469	0.197	0.814	−0.407	0.612
2	0.35	−7.145	0.791	−3.124	3.996	1.149	0.625	2.070	−0.959	1.700
3	0.45	−10.34	0.681	−5.054	6.785	1.946	1.565	3.777	−1.469	3.333
4	0.55	−13.18	−1.141	−6.819	9.933	2.450	3.886	5.606	−1.025	5.446
5	0.65	−15.40	−4.389	−7.509	13.23	2.532	7.534	6.839	1.117	7.144
6	0.75	−17.42	−9.193	−6.689	16.82	1.997	12.64	6.791	5.853	7.497
7	0.85	−26.49	−22.83	−9.682	28.95	1.808	27.84	9.366	21.02	14.38
8	0.95	−15.13	−14.87	1.409	18.73	−0.316	19.50	−0.715	17.82	−0.027

[a] $K_{Qz}' = iK_{Qy}'$.

Table 4 Harmonic coefficients at various radial positions for a single screw ship

Longitudinal wake velocities, $a_n = \frac{1}{\pi}\int_0^{2\pi} \frac{u}{U}(\theta)\cos n\theta d\theta$[a]

r/r_0	$a_0/2 - 1$ (mean − 1)	a_1	a_2	a_3	a_4	a_5	a_6	cosλ
0.25	−0.400	−0.075	−0.125	0.055	−0.037	0	−0.016	0.6212
0.35	−0.316	−0.091	−0.139	0.028	−0.046	0.010	−0.029	0.7427
0.45	−0.248	−0.105	−0.144	0.002	−0.051	0.016	−0.035	0.8189
0.55	−0.196	−0.113	−0.136	−0.021	−0.052	0.015	−0.035	0.8674
0.65	−0.168	−0.120	−0.127	−0.040	−0.050	0.002	−0.030	0.8996
0.75	−0.154	−0.127	−0.118	−0.052	−0.046	−0.012	−0.025	0.9219
0.85	−0.150	−0.132	−0.110	−0.061	−0.045	−0.023	−0.023	0.9375
0.95	−0.148	−0.136	−0.102	−0.067	−0.044	−0.030	−0.021	0.9490

Tangential velocity, $B_n = \frac{1}{\pi}\int_0^{2\pi} \frac{v}{u}(\theta)\sin n\theta d\theta$

r/r_0	B_0	B_1	B_2	B_3	B_4	B_5	B_6	−sinλ
0.25	0	−0.034	−0.037	0.055	−0.034	0.012	−0.017	−0.7837
0.35	0	−0.070	−0.040	0.033	−0.023	0.013	−0.019	−0.6695
0.45	0	−0.099	−0.044	0.015	−0.017	0.012	−0.019	−0.5740
0.55	0	−0.113	−0.048	0.002	−0.015	0.007	−0.017	−0.4976
0.65	0	−0.120	−0.051	−0.007	−0.014	0.002	−0.015	−0.4368
0.75	0	−0.120	−0.055	−0.013	−0.016	0	−0.013	−0.3875
0.85	0	−0.118	−0.057	−0.018	−0.018	−0.003	−0.011	−0.3480
0.95	0	−0.114	−0.060	−0.020	−0.020	−0.007	−0.011	−0.3151

[a] $\lambda = \text{ton}^{-1}U/\omega r$.

Table 5 Values of a_n cosλ and $-B_n$ sinλ[a]

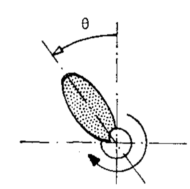
	a_n cosλ						
r/r_0	1 $q = 0$	2 $q = 1$	3 $q = 2$	4 $q = 3$	5 $q = 4$	6 $q = 5$	7 $q = 6$
0.25	−0.248	−0.047	−0.078	0.034	−0.023	0	−0.010
0.35	−0.235	−0.067	−0.103	0.021	−0.034	0.007	−0.021
0.45	−0.203	−0.086	−0.118	0.002	−0.042	0.013	−0.029
0.55	−0.170	−0.098	−0.118	−0.018	−0.045	0.013	−0.030
0.65	−0.151	−0.108	−0.114	−0.036	−0.045	0.002	−0.027
0.75	−0.142	−0.117	−0.109	−0.048	−0.042	−0.011	−0.023
0.85	−0.141	−0.124	−0.103	−0.057	−0.042	−0.022	−0.022
0.95	−0.140	−0.129	−0.097	−0.063	−0.042	−0.028	−0.020
	$-B_n$ sinλ, imaginary only						
	8	9	10	11	12	13	14
0.25	0	0.027	0.029	−0.043	0.027	−0.009	0.013
0.35	0	0.047	0.027	−0.022	0.015	−0.009	0.013
0.45	0	0.057	0.025	−0.009	0.010	−0.007	0.011
0.55	0	0.056	0.024	−0.001	0.007	−0.003	0.008
0.65	0	0.052	0.022	0.003	0.006	−0.001	0.006
0.75	0	0.047	0.021	0.005	0.006	0	0.005
0.85	0	0.041	0.020	0.006	0.006	0.001	0.004
0.95	0	0.036	0.019	0.006	0.006	0.002	0.003

[a] In general, the normal velocity is given by $W_N/U = (a_n \cos\lambda + A_n \sin\lambda) - i(b_n \cos\lambda + B_N \sin\lambda)$. Longitudinal component $= (a_n - ib_n)\cos\lambda$; tangential component $= (A_n - iB_n)\sin\lambda$.

Table 6 Values of $\tilde{K}_x(q)$ for $q = 5$[a]

Radial station ν	r/r_0	1 $\tilde{K}_x$ R (T2-2)	2 0.10^3 I (T2-3)	3 a_5 cosλ R (T5-6)	4 $-B_5$ sinλ I (T5-13)	5 1 × 3	6 2 × 4	7 5 − 6	8 1 × 4	9 2 × 3	10 8 + 9
1	0.25	−2.952	8.737	0	−0.009	0	−0.07863	0.07863	0.02657	0	0.02657
2	0.35	−4.851	18.82	0.007	−0.009	−0.03396	−0.1784	0.1444	0.04366	0.1317	0.1754
3	0.45	−4.106	29.97	0.103	−0.007	−0.05338	−0.2098	0.1564	0.02874	0.3896	0.4183
4	0.55	6.762	39.87	0.013	−0.003	0.08791	−0.1196	0.2075	−0.02029	0.5183	0.4980
5	0.65	25.75	43.77	0.002	−0.001	0.05150	−0.04377	0.09527	−0.02575	0.08754	0.06179
6	0.75	53.88	39.09	−0.011	0	−0.5927	0	−0.5927	0	−0.4300	−0.4300
7	0.85	133.9	56.63	−0.022	0.001	−2.946	0.05663	−3.003	0.1339	−1.2459	−1.1120
8	0.95	87.89	−8.236	−0.028	0.002	−2.461	−0.01647	−2.444	0.1758	0.2306	0.4064

[a] $\tilde{K}_x(5)_R \cdot 10^3 = -5.357$; $\tilde{K}_x(5)_I \cdot 10^3 = 0.0445$; magnitude $|\tilde{K}_x| = 0.005357$; phase $\phi = 179.5°$.



Blade angle θ = 36° for maximum force aft.

Table 7 Operator functions of blade bending moment coefficient, $M = BBM/\rho n^2D^5$, about 0.25 radius due to wake only, of unit amplitude

		1	2	3	4	5	6	7	8	9	10	11	12	13	14
		$q = 0$		$q = 1$		$q = 2$		$q = 3$		$q = 4$		$q = 5$		$q = 6$	
ν	r/r_0	R	I	R	I	R	I	R	I	R	I	R	I	R	I
1	0.25	−0.00042	0	0.00029	0.00005	0.00017	0.00008	0.00009	0.00009	0.00005	0.00010	0.00001	0.00012	−0.00002	0.00009
2	0.35	−0.00104	0	0.00081	0.00021	0.00057	0.00034	0.00035	0.00038	0.00015	0.00039	−0.00003	0.00039	−0.00015	0.00033
3	0.45	−0.00185	0	0.00155	0.00042	0.00119	0.00071	0.00080	0.00081	0.00037	0.00083	−0.00003	0.00082	−0.00030	0.00074
4	0.55	−0.00279	0	0.00241	0.00058	0.00198	0.00107	0.00153	0.00130	0.00095	0.00134	0.00032	0.00133	−0.00024	0.00131
5	0.65	−0.00279	0	0.00334	0.00063	0.00289	0.00127	0.00248	0.00166	0.00191	0.00171	0.00113	0.00172	0.00029	0.00179
6	0.75	−0.00491	0	0.00434	0.00051	0.00393	0.00118	0.00366	0.00168	0.00326	0.00175	0.00261	0.00180	0.00151	0.00193
7	0.85	−0.00826	0	0.00757	0.00049	0.00737	0.00124	0.00736	0.00201	0.00726	0.00247	0.00696	0.00303	0.00548	0.00377
8	0.95	−0.00516	0	0.00492	−0.00008	0.00487	−0.00001	0.00500	0.00001	0.00512	−0.00018	0.00500	−0.00034	0.00467	0

where the table column designations apply to the application to a 5-bladed propeller.

The vertical force is similar in structure, being given by

$$\tilde{K}_z(q) = \sum_{\nu=1}^{8} \{\tilde{K}_{z\nu}'(q-1)w_{q-1}(\nu) - \tilde{K}_{z\nu}(q+1)w_{q+1}(\nu)\} \quad (22)$$

and using
$$\tilde{K}_z' = -i\tilde{K}_y' \quad (23)$$

$$\tilde{K}_z(q) = -i \sum_{\nu=1}^{8} [\tilde{K}_{y\nu}(q-1)w_{q-1}(\nu)] + [\tilde{K}_{y\nu}(q+1)w_{q+1}(\nu)] \quad (24)$$

$$\therefore \tilde{K}_z(q) = [\tilde{K}_y(q)_I - i(\tilde{K}_y(q)]_R \quad (25)$$

Hence Eq. (25) may be evaluated using the indicated terms from Eqs. (21) and (20).

Formulas for the moments $\tilde{K}_{Q_y}$ and $\tilde{K}_{Q_z}$ are similar with corresponding columns employed from Table 3.

The operators for blade bending moment about the 0.25 radius for the 5-bladed propeller are given in Table 7. It is to be noted that each blade feels all the wake harmonics and, hence, the bending moment is composed of the sum of all spatial frequencies. Thus, it is necessary to calculate the variation in moment for each frequency (0–7) for a complete cycle of a blade and to add these variations to secure the maximum and minimum of the blade bending moment.

The expression for the dimensionless blade bending moment is

$$\tilde{K}_M = \text{real part} \sum_{q=0}^{7} \left\{ \sum_{\nu=1}^{8} [K_{M\nu}'(q)w_q(\nu)]e^{iq\theta} \right\}$$

or

$$\tilde{K}_M = \sum_{q=0}^{7} (\{[K_{M\nu}'(q)]_R a_q \cos\lambda + [K_{M\nu}'(q)]_I B_q \sin\lambda\}\cos q\theta + [K_{M\nu}'(q)]_R B_q \sin\lambda - [K_{M\nu}'(q)]_I a_q \cos\lambda\} \sin q\theta)$$

The real values of $K_M'(q)$ are to be found in the odd-numbered columns of Table 7. The imaginary ones are given in the even-numbered columns. Curves of the various contributions, as well as a curve of their sum, is shown in Fig. 1 for the specific case of a 5-bladed propeller in the wake of a single screw hull.

Use of the Operator Concept to Minimize Propeller Forces

The fact that propeller forces are the net of the sum of products of local operator coefficients and local wake values immediately raises the prospect of configuring the chord distribution and, hence, the operator distribution to yield a minimum sum or force with a given (known) radial distribution of wake Such a minimization must be carried out subject to the constraint imposed by the requirement of meeting a desired thrust and torque to yield an efficient propeller and, in addition, the load distribution must be such to avoid cavitation. For example, fitting a 4-bladed propeller to a single screw ship will provide small transverse and vertical forces (as these depend upon the 3rd and 5th harmonics of the wake which are generally weak in this class of vessel), but the thrust and torque variations may be expected to be large because the fourth harmonic is generally strong. However, if the blade chord and pitch could be taken to minimize the thrust and torque, then the adaptation of a 4-bladed propeller would be attractive.

It is of interest to observe the relative variations of the axial force operation for $q = 5$ as displayed in Fig. 2. It is seen that the real or inphase component is extremely small for radial positions less than 0.6, whereas the imaginary part (out-of-phase) is relatively larger in that range and becomes relatively smaller beyond 0.75. This behavior is due to the fact that the inner radial elements must negotiate the same number of variations in a small circumference and, hence, operate at higher reduced frequencies than those elements out near the tip of the blade. Specifically, the reduced frequency k at any radial section is

$$k(r/r_0) = q\gamma/\{1 + [(r_0/r)(J/\pi)]^2\}^{1/2}$$

where $\gamma = \tan^{-1}(c/2/r)$, angle subtended by semichord at propeller axis as shown in Fig. 2.

A plot of the local reduced frequency for the blades of the 5-bladed propeller, which is given in Fig. 2, shows that the inner radial positions have somewhat higher values than those at outer positions. Thus, one may expect the real or inphase

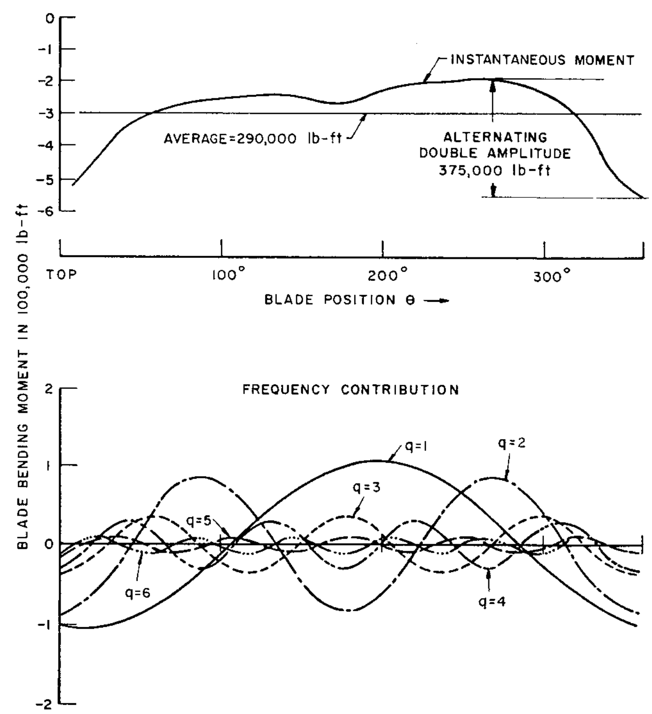


Fig. 1 Blade bending moment, due to propeller loading, about pitch line at 0.3 radius, on 5-bladed propeller abaft a single screw ship.

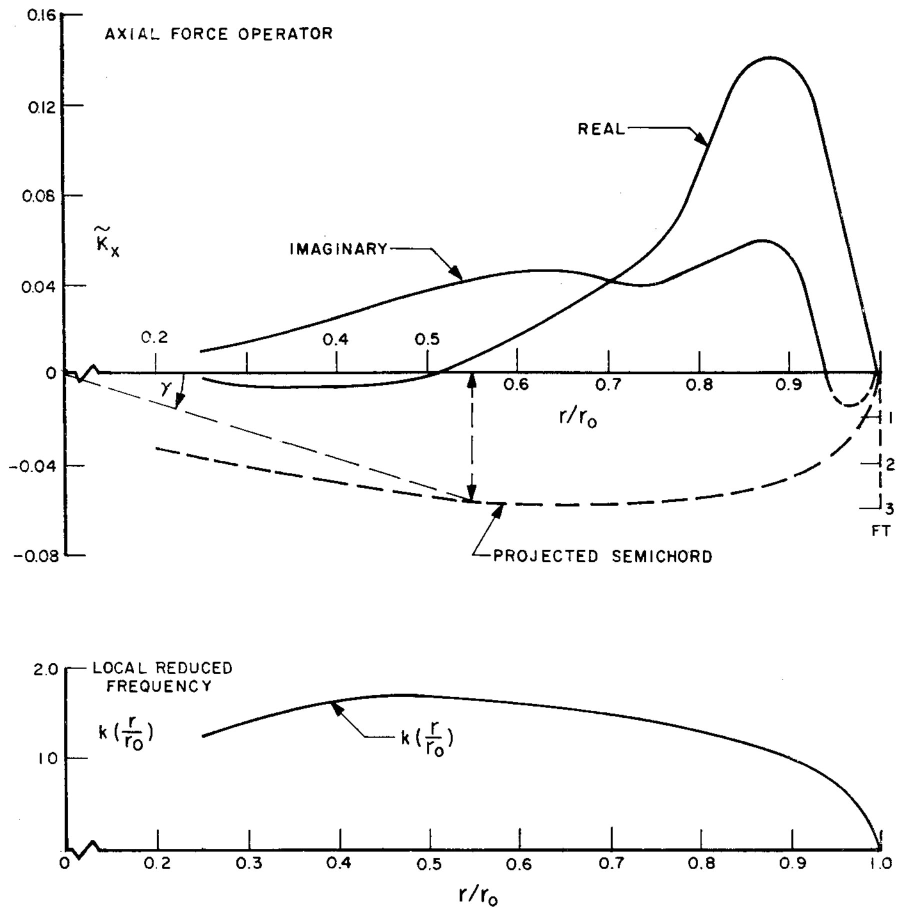


Fig. 2 Coefficient form of axial force operator function for a 5-bladed propeller.

operator to be smaller at the inner radii because of the degradation of circulatory loading at higher reduced frequencies and the opposite behavior for the imaginary or out-of-phase, which is a function mainly of the relative acceleration of the flow as seen by the blade elements. Thus, it may be possible to control the distribution of these components by distributing the chord of the blade to give operators that will mismatch with a given wake. This remains to be investigated by application of the program.

Summary and Recommendations

The foregoing development and numerical results demonstrate that force and moment operators can be found from an existing computer program which has been validated for effective prediction of mean and vibratory forces and moments generated by propellers in spatially variable wakes or inflow. Thus, any propeller can be completely characterized and the resulting force operators may be used with any given wake to give mean and vibratory excitations through the use of a desk calculator in a matter of a few minutes.

It is recommended that this method be applied to completely characterize an entire family of propellers over the practical range of blade area ratio and pitch-diameter ratio so that prediction of mean and vibratory excitations may be made by any naval architect for his particular applications in a matter of minutes without the use of a computer. These results could then be analyzed to provide a knowledge of the dependence of these operators on blade area ratio, pitch-diameter ratio and number of blades. In addition, the method should be applied to families of unusual blade outlines as well as to skewed blades to ascertain a method of designing a propeller which will have minimal excitations and yet fulfill practical constraints.

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